The double of $U_{\eta,\gamma}$

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(170513) $\mathcal{U}_{\eta,\gamma}$ conventions: $A = \langle g, G = e^{\eta g}, e\rangle/([g,e] = \gamma e)$ with $\Delta(g,G,e) = (g_1 + g_2, G_1G_2, e_1G_2 + e_2)$; $S(g,G,e) = (-g, G^{-1}, -eG^{-1})$ and dual $A^* = \langle h, H = e^{\gamma h}, f\rangle/([h,f] = -\eta h)$ with $\Delta(h,H,f) = (h_1+h_2, H_1H_2, f_1 + H_1 f_2)$; $S(h,H,f) = (-h, H^{-1}, -H^{-1}f)$. Pairing by $(g,e)^* = (h,f)$.

(160611) The quantum double $\mathcal{D}A :=$  $A^{*,op} \otimes A$ with $(\phi a)(\psi b) := \langle S a_1, \psi_1\rangle\langle a_3, \psi_3\rangle(\psi_2\phi)(a_2 b)$. What problem does it solve?

$$gh = \langle Sg_1, h_1\rangle\langle g_3, h_3\rangle h_2 g_2 =$$

$$= \langle -g, h_1\rangle\langle 1, h_3\rangle h_2 1 + \langle 1, h_1\rangle\langle 1, h_3\rangle h_2 g + \langle 1, h_1\rangle\langle g, h_3\rangle h_2 1$$

$$= -h_2 + hg + h_2 = hg$$

$$gf = \langle -g, f_1\rangle\langle 1, f_3\rangle f_2 1 + \langle 1, f_1\rangle\langle 1, f_3\rangle f_2 g + \langle 1, f_1\rangle\langle g, f_3\rangle f_2$$

$$= -\gamma f + fg + 0$$